1 Gravitating Particles in a Box

An ideal gas of N particles of mass m is confined to a cubic box of volume L^3 at temperature T. The box is in a uniform gravitational field whose potential energy is $U(y) = mgy$. Write down the partition function for the system as an integral over position and momentum coordinates. Find the energy and heat capacity of the system.

(Hint: The partition function should be a dimensionless normalizer ... be careful.)

2 Diffusion and Drift

Suppose you have particles of density ρ and current density J whose total number is conserved. The particles experience both diffusion and drift in one dimension so that the current density J is given by $\rho v - D\partial_x \rho$, where D is the diffusion constant.

Write down the conservation equation for these particles and examine the steady state where $\rho(x,t) \to \rho(x)$. Solve for $\rho(x)$ for arbitrary velocity $v(x)$ (you may leave your answer in terms of an integral). Next consider the case of of a particle experiencing drag, so that $v = -\frac{1}{\zeta}\frac{dU}{dx}$, and solve for $\rho(x)$ in terms of $U(x)$.

(Problem pirated from Chris Wiggins.)

3 Fermi Gas

Consider a gas of N non-interacting electrons at temperature $T = 0$ in a cubic box of volume L^3. Find the Fermi energy of the system.